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Viscous Fluid Cosmology in Bianchi Type-I Space-Time

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Abstract The paper presents a spatially homogeneous and anisotropic Bianchi type-I cosmological model consisting of a dissipative fluid. The field equations are solved explicitly by using a law of variation for mean Hubble parameter, which is related to average scale factor and yields a constant value for deceleration parameter. We find that the constant value of deceleration parameter describes the different phases of the evolution of universe. A barotropic equation of state ($p=\gamma\rho$) together with a linear relation between shear viscosity and expansion scalar, is assumed. It is found that the viscosity plays a key role in the process of the isotropization of the universe. The presence of viscous term does not change the fundamental nature of initial singularity. The thermodynamical properties of the solutions are studied and the entropy distribution is also given explicitly.

Keywords Hubble parameter · Deceleration parameter · Cosmological models · Viscous fluid

1 Introduction

In recent years the introduction of viscosity in the cosmic fluid content has been found useful in explaining many important physical aspects of the dynamics of homogeneous cosmological models. The dissipative mechanisms not only modify the nature of the singularity, usually occurring for a perfect fluid, but also can successfully account for the large entropy per baryon in the present universe. In fact the observed physical phenomena such as the large entropy per baryon and the remarkable degree of the isotropy of cosmic microwave background radiation (CMBR) reveal the importance of analysis of the dissipative effects in cosmology. The physical processes such as the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era are expected

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to give rise to viscous effects. Moreover according to the Grand Unified Theory (GUT), the phase transition and string creation are also believed to involve viscous effects. Therefore it makes sense to consider viscous cosmological models.

Misner [1, 2], Weinberg [3], Nightingale [4], Murphy [5], Heller and Klimek [6] are some of the authors who have investigated the role of viscosity in avoiding the initial big bang singularity. On the other hand Belinskii and Khalatnikov [7] have studied the nature of cosmological solutions of a homogeneous Bianchi type-I model by taking into account the dissipative process due to viscosity and showed that viscosity can not remove the initial big bang singularity rather it can cause qualitatively new behavior of the solutions near the singularity. Banerjee et al. [8, 9], Barrow [10], Grøn [11], Gavrilov et al. [12], Arbab [13], Beesham et al. [14], Krori and Mukherjee [15] and many others have studied viscous cosmological models in different contexts. Grøn [11] has reviewed viscous cosmological models and deduced that viscosity plays an important role in the process of isotropization of the universe. Saha [16, 17] has investigated the role of viscosity together with a spinor field in the evolution of Bianchi type-I universe. Singh and Chaubey [18] have considered bulk and shear viscosity in Bianchi-V universe. Bali and Singh [19] have studied Bianchi-I space-time with bulk viscosity and variable cosmological constant.

In this paper our intention is to study some physically realistic and exact viscous Bianchi type-I models in general relativity. Therefore, in what follows, we consider a spatially homogeneous and anisotropic Bianchi-I space-time model filled with perfect fluid containing the bulk and shear viscosity content. This work is organized as follows: The model and field equations are given in Sect. 2. The field equations are solved in Sect. 3 by assuming some physically relevant assumptions. Two exact Bianchi type-I models are presented in Sects. 3.1 and 3.2 and physical behavior of the models is explained in each section. Some thermodynamical parameters of the models are obtained in Sect. 4 and the concluding remarks are presented in Sect. 5.

2 Model and Field Equations

The spatially homogeneous and anisotropic Bianchi-I space-time is described by the line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \tag{1}$$

where $A(t)$, $B(t)$ and $C(t)$ are the metric functions of cosmic time t.

We define $a = (ABC)^{\frac{1}{3}}$ as the average scale factor of Bianchi type-I universe so that the generalized Hubble parameter in anisotropic models may be defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{2}$$

where an over dot denotes derivative with respect to the cosmic time t.

Also we have

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{3}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the directions of x-, y- and z-axes respectively.

In the system of units $8\pi G = c = 1$, the Einstein's field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij}, \tag{4}$$

where T_{ij} is the stress energy tensor of matter which, in case of viscous fluid, has the form [20]

$$T_{ij} = (\rho + \bar{p}) u_i u_j + \bar{p} g_{ij} - \eta \mu_{ij}, \tag{5}$$

with

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta\right) u^i_{;i} = p - (3\xi - 2\eta) H \tag{6}$$

and

$$\mu_{ij} = u_{i;j} + u_{j;i} + u_i u^\alpha u_{j;\alpha} + u_j u^\alpha u_{i;\alpha}. \tag{7}$$

In the above equations ξ and η stand for the bulk and shear viscosity coefficients, respectively; ρ is the matter density; p is the isotropic pressure and u^i is the four-velocity vector satisfying $u^i u_i = -1$.

In a co-moving coordinate system, where $u^i = \delta^i_0$, the field equations (4), for the anisotropic Bianchi type-I space-time (1) and viscous fluid distribution (5), yield

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\bar{p} + 2\eta \frac{\dot{A}}{A}, \tag{8}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\bar{p} + 2\eta \frac{\dot{B}}{B}, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\bar{p} + 2\eta \frac{\dot{C}}{C}, \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \rho. \tag{11}$$

The usual definitions of the dynamical scalars such as the expansion scalar (θ) and the shear scalar (σ) are considered to be

$$\theta = u^i_{;i} = \frac{3\dot{a}}{a} \tag{12}$$

and

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2, \tag{13}$$

where

$$\sigma_{ij} = u_{i;j} + \frac{1}{2}\left(u_{i;k} u^k u_j + u_{j;k} u^k u_i\right) + \frac{1}{3}\theta(g_{ij} + u_i u_j). \tag{14}$$

The anisotropy parameter $(\bar{A})$ is defined as

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{15}$$

The energy conservation equation $T^{ij}_{;j} = 0$, leads to

$$\dot{\rho} = -(\rho + p)\theta + \xi\theta^2 + 4\eta\sigma^2. \tag{16}$$

It follows from (16) that for contraction, that is, $\theta < 0$, we have $\dot{\rho} > 0$ so that the matter density increases or decreases depending on whether the viscous heating is greater or less than the cooling due to expansion.

Now the Raychaudhuri equation [21] is given by

$$\dot{\theta} = -2\sigma^2 - \frac{1}{3}\theta^2 + R_{ij}u^i u^j, \tag{17}$$

where

$$R_{ij}u^i u^j = -\frac{1}{2}[\rho + 3(p - \xi\theta)]. \tag{18}$$

The Hawking-Penrose energy condition $R_{ij}u^i u^j \leq 0$, implies that $\rho + 3p \geq 3\xi\theta$. It follows that $\dot{\theta} < 0$ and there can be only maximum and no minimum for the expansion, which implies that a singularity is unavoidable.

We have a system of four independent equations (8)–(11) and seven unknown variables, namely A, B, C, p, ρ, ξ and η. So for complete determinacy of the system, we need three appropriate relations among these variables that we shall consider in the following section and solve the field equations.

3 Solution of Field Equations

We follow the approach of Saha [16] to solve the field equations (8)–(11). Subtracting (8) from (9), (8) from (10), (9) from (10) and taking second integral of each, we get the following three relations respectively:

$$\frac{A}{B} = d_1 \exp\left(x_1 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{19}$$

$$\frac{A}{C} = d_2 \exp\left(x_2 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{20}$$

$$\frac{B}{C} = d_3 \exp\left(x_3 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{21}$$

where d_1, x_1, d_2, x_2, d_3 and x_3 are constants of integration.

From (19)–(21), the metric functions can be explicitly written as

$$A(t) = a_1 a \exp\left(b_1 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{22}$$

$$B(t) = a_2 a \exp\left(b_2 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{23}$$

$$C(t) = a_3 a \exp\left(b_3 \int a^{-3} e^{-2\int \eta dt} dt\right), \tag{24}$$

where

$$a_1 = \sqrt[3]{d_1 d_2}, \qquad a_2 = \sqrt[3]{d_1^{-1} d_3}, \qquad a_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$

$$b_1 = \frac{x_1 + x_2}{3}, \qquad b_2 = \frac{x_3 - x_1}{3}, \qquad b_3 = \frac{-(x_2 + x_3)}{3}.$$

These constants satisfy the following two relations:

$$a_1 a_2 a_3 = 1, \qquad b_1 + b_2 + b_3 = 0. \tag{25}$$

Thus the metric functions are found explicitly in terms of the average scale factor a.

For any physically relevant model, the Hubble parameter and deceleration parameter (DP) are the most important observational quantities. The first quantity sets the present time scale of the expansion while the second one reveals that the present state of evolution of universe is speeding up instead of slowing down as expected before the type Ia supernovae observations. Therefore we consider that the generalized mean Hubble parameter H is related to the average scale factor a by the relation

$$H = Da^{-n}, \tag{26}$$

where $D \geq 0$ and $n > 0$ are constants. Equation (26) yields a constant value for the DP as we shall see in what follows. This type of relation has already been considered by Berman [22], and Berman and Gomide [23] for solving Friedmann-Robertson-Walker (FRW) models. Later on several authors (see, Singh and Kumar [24] and references therein) have considered FRW cosmological models with constant DP. Singh and Kumar [24–27] and Kumar and Singh [28–30] have extended this work to anisotropic Bianchi type-I and II cosmological models in general relativity and some scalar tensor theories of gravitation with constant DP.

The DP (q) is defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{27}$$

From (2) and (26), we get

$$\dot{a} = Da^{-n+1}, \tag{28}$$

$$\ddot{a} = -D^2(n-1)a^{-2n+1}. \tag{29}$$

Integration of (28) gives

$$a = (nDt)^{\frac{1}{n}} \quad \text{for } n \neq 0, \tag{30}$$

and

$$a = C_0 e^{Dt} \quad \text{for } n = 0, \tag{31}$$

where C_0 is a constant of integration.

Substituting (28) and (29) into (27), we get

$$q = n - 1. \tag{32}$$

This shows that the value of DP is a constant. Here in (30), we have assumed that $a = 0$ at $t = 0$ so that the constant of integration turns out to be zero. Thus we have derived two types of cosmology depending on whether $n \neq 0$ or $n = 0$. It is however possible to have $D = 0$ in (26) for which we would have a static universe. But $D > 0$ is consistent with the observations which favor the expanding universe. The sign of q indicates whether the model inflates or not. A positive sign of q i.e. $n > 1$ corresponds to the standard decelerating model whereas the negative sign $-1 \leq q < 0$ indicates the inflation. Recent observations show that the DP of the universe is in the range $-1 \leq q < 0$ and the present day universe is undergoing an accelerated expansion (Perlmutter et al. [31–33], Riess et al. [34, 35], Tonry et al. [36], Knop et al. [37] and John [38]). It may be noted that though the current observations of SNe Ia and the CMBR favor an accelerating model ($q < 0$), they do not altogether rule out the existence of decelerating phase in early history of our universe [39]. The present day universe has been thought as the Einstein-de Sitter with $q = \frac{1}{2}$. Since the recent observations of supernovae data confirm that the universe is accelerating, we may ask whether the value of DP could be different from the de Sitter universe, say as defined in (32). In case of accelerating universe, the second case ($n = 0$) becomes very relevant to describe the physical nature of the expanding universe.

Next, we assume that the coefficient of shear viscosity (η) is proportional to the expansion scalar (θ) i.e. $\eta \propto \theta$, which leads to

$$\eta = \eta_0 \theta, \tag{33}$$

where η_0 is proportionality constant. Such relation has already been proposed in the physical literature as a physically plausible relation [18, 40, 41].

Finally to conveniently specify the source, we assume the perfect gas equation of state, which may be written as

$$p = \gamma \rho, \quad 0 \leq \gamma \leq 1. \tag{34}$$

In the following sections we present exact Bianchi-I models for $n \neq 0$ and $n = 0$, respectively.

3.1 Cosmology for $n \neq 0$

Using (30) and (33) into (22)–(24), we get the following expressions for scale factors:

$$A(t) = a_1 (nDt)^{\frac{1}{n}} \exp\left[\frac{b_1}{D(n - 6\eta_0 - 3)} (nDt)^{\frac{n-6\eta_0-3}{n}}\right], \tag{35}$$

$$B(t) = a_2 (nDt)^{\frac{1}{n}} \exp\left[\frac{b_2}{D(n - 6\eta_0 - 3)} (nDt)^{\frac{n-6\eta_0-3}{n}}\right], \tag{36}$$

$$C(t) = a_3 (nDt)^{\frac{1}{n}} \exp\left[\frac{b_3}{D(n - 6\eta_0 - 3)} (nDt)^{\frac{n-6\eta_0-3}{n}}\right]. \tag{37}$$

The physical parameters such as directional Hubble factors (H_i), Hubble parameter (H), expansion scalar (θ), spatial volume (V), anisotropy parameter ($\bar{A}$) and shear scalar (σ) are given by

$$H_i = (nt)^{-1} + b_i (nDt)^{\frac{-6\eta_0-3}{n}}. \tag{38}$$

$$H = (nt)^{-1}, \tag{39}$$

$$\theta = 3(nt)^{-1}, \tag{40}$$

$$V = (nDt)^{\frac{3}{n}}, \tag{41}$$

$$\bar{A} = \frac{1}{3D^2}(b_1^2 + b_2^2 + b_3^2)(nDt)^{\frac{2n-12\eta_0-6}{n}}, \tag{42}$$

$$\sigma^2 = \beta(nDt)^{\frac{-12\eta_0-6}{n}}, \tag{43}$$

where $\beta = b_1^2 + b_2^2 + b_1 b_2$.

The effective pressure and energy density of the model read as

$$\bar{p} = (2n - 6\eta_0 - 3)(nt)^{-2} - \beta(nDt)^{\frac{-12\eta_0-6}{n}}, \tag{44}$$

$$\rho = 3(nt)^{-2} - \beta(nDt)^{\frac{-12\eta_0-6}{n}}. \tag{45}$$

The expressions for isotropic pressure, bulk viscosity and shear viscosity are given by

$$p = 3\gamma(nt)^{-2} - \gamma\beta(nDt)^{\frac{-12\eta_0-6}{n}}, \tag{46}$$

$$\xi = \frac{D[3(1+\gamma) - 2n]}{3}(nDt)^{-1} + \frac{(1-\gamma)\beta}{3D}(nDt)^{\frac{n-12\eta_0-6}{n}}, \tag{47}$$

$$\eta = 3\eta_0(nt)^{-1}. \tag{48}$$

One may observe that the above set of solutions satisfy the energy conservation equation (16) identically. Therefore the above solutions are exact solutions of the Einstein's field equations (8)–(11). It is observed that the spatial volume is zero at $t = 0$ and expansion scalar is infinite, which shows that the universe starts evolving with zero volume at $t = 0$ with a big bang. The scale factors also vanish at $t = 0$ and hence the model has a point singularity at the initial epoch. The pressure, energy density, bulk viscosity, shear viscosity, Hubble factors and shear scalar diverge at $t = 0$. The anisotropy parameter also tends to infinity at the initial epoch provided $n < 6\eta_0 + 3$. The universe exhibits the power-law expansion after the big bang impulse. As t increases, the scale factors and spatial volume increase but the expansion scalar decreases. The physical requirement that ξ and η are positive, is fulfilled provided $3(1+\gamma) > 2n$ and $\eta_0 > 0$. Thus in an expanding model, the viscosity coefficients, which have infinitely large magnitudes near the initial singular state, monotonically decrease with time. Also the magnitude of shear as well as the expansion rate diminish in the course of expansion. As $t \to \infty$, scale factors and volume become infinite whereas ρ, p, ξ, η, H_1, H_2, H_3, θ, $\bar{A}$ and σ^2 tend to zero. Therefore the model would essentially give an empty universe for large times t.

From (40) and (43), we get

$$\frac{\sigma^2}{\theta^2} = \frac{\beta}{9D^2}(nDt)^{\frac{-12\eta_0-6+2n}{n}}. \tag{49}$$

This shows that the ratio of shear and expansion scalars decays to zero as $t \to \infty$ provided $n < 6\eta_0 + 3$. So the model approaches to isotropy for large values of t. Also we observe that the rate of decay falls in the absence of shear viscosity. From (40) and (45), one can get the following relation:

$$\frac{\rho}{\theta^2} = \frac{1}{3} - \frac{\beta}{9D^2}(nDt)^{\frac{-12\eta_0-6+2n}{n}}, \tag{50}$$

which shows that $\frac{\rho}{\theta^2}$ is maximum at $t = \infty$ and the maximum value is $\frac{1}{3}$.

It is interesting to note that the anisotropy parameter decreases faster with time due to the presence of viscosity. Therefore the solutions reveal that viscosity has played an important role in the process of isotropization of the large scale structure of the universe. Finally, we conclude that the model represents shearing, non-rotating and expanding model of the universe, which starts with a big bang and approaches to isotropy at late times.

3.2 Cosmology for $n = 0$

Using (31) and (33) into (22)–(24), the scale factors of the model read as

$$A(t) = a_1 C_0 \exp\left[Dt - \frac{b_1}{3D(2\eta_0 + 1)C_0^3} e^{-3(2\eta_0+1)Dt}\right], \tag{51}$$

$$B(t) = a_2 C_0 \exp\left[Dt - \frac{b_2}{3D(2\eta_0 + 1)C_0^3} e^{-3(2\eta_0+1)Dt}\right], \tag{52}$$

$$C(t) = a_3 C_0 \exp\left[Dt - \frac{b_3}{3D(2\eta_0 + 1)C_0^3} e^{-3(2\eta_0+1)Dt}\right]. \tag{53}$$

The other physical parameters of the model have the following expressions:

$$H_i = D + b_i C_0^{-3} e^{-3(2\eta_0+1)Dt}, \quad (i = 1, 2, 3) \tag{54}$$

$$H = D, \tag{55}$$

$$\theta = 3D, \tag{56}$$

$$V = C_0^3 e^{3Dt}, \tag{57}$$

$$\bar{A} = \frac{1}{3D^2}(b_1^2 + b_2^2 + b_3^2) C_0^{-6} e^{-6(2\eta_0+1)Dt}, \tag{58}$$

$$\sigma^2 = C_0^{-6} \beta e^{-6(2\eta_0+1)Dt}. \tag{59}$$

The effective pressure, energy density, isotropic pressure, bulk viscosity and shear viscosity read as

$$\bar{p} = (6\eta_0 - 3)D^2 - \beta C_0^{-6} e^{-6(2\eta_0+1)Dt}, \tag{60}$$

$$\rho = 3D^2 - \beta C_0^{-6} e^{-6(2\eta_0+1)Dt}, \tag{61}$$

$$p = 3\gamma D^2 - \gamma\beta C_0^{-6} e^{-6(2\eta_0+1)Dt}, \tag{62}$$

$$\xi = (1 + \gamma)D + \frac{(1 - \gamma)\beta C_0^{-6}}{3D} e^{-6(2\eta_0+1)Dt}, \tag{63}$$

$$\eta = 3\eta_0 D. \tag{64}$$

We find that the above set of solutions satisfy (16) identically. The model has no initial singularity. The spatial volume, scale factors, pressure, energy density, shear viscosity, bulk viscosity and the other cosmological parameters are constants at $t = 0$. Thus the universe starts with a non-singular state at $t = 0$. The viscosity coefficients start with finite magnitudes at the initial epoch. As t increases, the scale factors and the spatial volume increase exponentially while bulk viscosity decreases. The anisotropy parameter decreases faster due to the viscosity effects. Therefore viscosity has played a key role in the process of isotropization of universe. It is interesting to note that the expansion scalar is constant throughout the evolution of universe and therefore the universe exhibits uniform exponential expansion in this model. As $t \to \infty$, the scale factors and volume of the universe become infinitely large whereas the anisotropy parameter and shear scalar tend to zero. The pressure, energy density, shear viscosity, bulk viscosity and Hubble factors become constants.

The ratio of shear and expansion scalars is given by

$$\frac{\sigma^2}{\theta^2} = \frac{\beta C_0^{-6}}{9D^2} e^{-6(2\eta_0+1)Dt}, \tag{65}$$

which decays exponentially and the rate of decay falls in the absence of shear viscosity. It tends to zero as $t \to \infty$, which implies that the model approaches to isotropy at late times.

The relation

$$\frac{\rho}{\theta^2} = \frac{1}{3} - \frac{\beta C_0^{-6}}{9D^2} e^{-6(2\eta_0+1)Dt}, \tag{66}$$

indicates that if one starts with zero mass density at some finite time, one must have $(\frac{\rho}{\theta^2})_{max} = \frac{1}{3}$ as $t \to \infty$.

Hence the model represents a shearing, non-rotating and expanding universe with a finite start approaching to isotropy at late times.

4 Thermodynamical Equations

The energy in a comoving volume is $U = \rho V$. The equation for production of entropy S in a comoving volume due to the dissipative effects in a fluid with temperature T is given by

$$T\dot{S} = \dot{U} + p\dot{V} = 3V(3\xi + 2\eta\bar{A})H^2. \tag{67}$$

In a cosmic fluid where the energy density and pressure of the cosmic fluid are functions of temperature only, $\rho = \rho(T)$, $p = p(T)$ and where the cosmic fluid has no net charge, we obtain easily (Grøn [11])

$$S = \frac{V}{T}(\rho + p). \tag{68}$$

From (67) and (68), we get the following expression for the entropy production rate in viscous Bianchi type-I universe:

$$\frac{\dot{S}}{S} = \frac{3(3\xi + 2\eta\bar{A})H^2}{\rho + p}. \tag{69}$$

For a fluid obeying the equation of state (34), (68) and (69) become

$$S = \frac{V}{T}(1+\gamma)\rho, \tag{70}$$

$$\frac{\dot{S}}{S} = \frac{3(3\xi + 2\eta\bar{A})H^2}{(1+\gamma)\rho}. \tag{71}$$

Equation (71) can be rewritten as

$$\frac{\dot{S}}{S} = \frac{\xi + 4\eta(\sigma^2/\theta^2)}{(1+\gamma)(\rho/\theta^2)}, \tag{72}$$

which gives the rate of change of entropy with time. Clearly $\frac{\dot{S}}{S} > 0$ as long as viscosity coefficients are positive since $\sigma^2/\theta^2 > 0$ and $\rho/\theta^2 > 0$. Hence $\dot{S} > 0$, which implies that the total entropy always increases with the change of proper time irrespective of an expanding or contracting model. Also we observe that $\frac{\dot{S}}{S} \to \infty$ as $\frac{\rho}{\theta^2} \to 0$, and the universe becomes homogeneous.

Let the entropy density be s so that

$$s = \frac{S}{V} = \frac{(1+\gamma)\rho}{T}. \tag{73}$$

It defines the entropy density in terms of the temperature.

The first law of thermodynamics may be written as

$$d(\rho V) + \gamma\rho dV = (1+\gamma)Td\left(\frac{\rho V}{T}\right), \tag{74}$$

which on integration, yields

$$T \sim \rho^{\frac{\gamma}{1+\gamma}}. \tag{75}$$

From (73) and (75), one can get

$$s \sim \rho^{\frac{1}{1+\gamma}}. \tag{76}$$

The entropy in a comoving volume then varies according to

$$S \sim sV. \tag{77}$$

These equations are not valid for a vacuum fluid with $\gamma = -1$. For a Zel'dovich fluid $(\gamma = 1)$, we get

$$T \sim \rho^{\frac{1}{2}} \quad \text{and} \quad s \sim \rho^{\frac{1}{2}} \sim T,$$

so that the entropy density is proportional to the temperature.

For $n \neq 0$, we find the respective temperature, entropy density and total entropy from (75)–(77), as

$$T = T_0\left[3(nt)^{-2} - \beta(nDt)^{\frac{-12\eta_0-6}{n}}\right]^{\frac{\gamma}{1+\gamma}}, \tag{78}$$

$$s = s_0\left[3(nt)^{-2} - \beta(nDt)^{\frac{-12\eta_0-6}{n}}\right]^{\frac{1}{1+\gamma}}, \tag{79}$$

$$S = S_0(nDt)^{\frac{3}{n}}\left[3(nt)^{-2} - \beta(nDt)^{\frac{-12\eta_0-6}{n}}\right]^{\frac{1}{1+\gamma}}, \tag{80}$$

whereas in case of $n = 0$, these parameters are given by

$$T = T_{00}[3D^2 - \beta C_0^{-6}e^{-6(2\eta_0+1)Dt}]^{\frac{\gamma}{1+\gamma}}, \tag{81}$$

$$s = s_{00}[3D^2 - \beta C_0^{-6}e^{-6(2\eta_0+1)Dt}]^{\frac{1}{1+\gamma}}, \tag{82}$$

$$S = S_{00}e^{3Dt}[3D^2 - \beta C_0^{-6}e^{-6(2\eta_0+1)Dt}]^{\frac{1}{1+\gamma}}, \tag{83}$$

where T_0, s_0, S_0, T_{00}, s_{00} and S_{00} are positive constants. For $n \neq 0$, it is observed that the total entropy $S = 0$ at $t = \frac{1}{nD}[\frac{\beta}{3D^2}]^{\frac{n}{2(6\eta_0+3-n)}}$, where we also have $\frac{\rho}{\theta^2} = 0$. Again as $t \to \infty$; $S \to \infty$ and $\frac{\rho}{\theta^2}$ approaches its maximum value. For $n = 0$, the total entropy never vanishes and goes on increasing as the evaluation progresses.

Entropy change For $n \neq 0$, (71) leads to

$$\frac{\dot{S}}{S} = \frac{3[3(1+\gamma) - 2n](nt)^{-3} + 3D(1-\gamma+4\eta_0)\beta(nDt)^{\frac{-12\eta_0-6}{n}-1}}{(1+\gamma)[3(nt)^{-2} - \beta(nDt)^{\frac{-12\eta_0-6}{n}}]}, \tag{84}$$

whereas in case of $n = 0$, we get

$$\frac{\dot{S}}{S} = \frac{9(1+\gamma)D^3 + 3D(1-\gamma+4\eta_0)\beta e^{-6(2\eta_0+1)Dt}}{(1+\gamma)[3D^2 - \beta C_0^{-6} e^{-6(2\eta_0+1)Dt}]}. \tag{85}$$

It is observed from these relations that $\frac{\dot{S}}{S} > 0$, which implies that the total entropy increases with time in the Bianchi-I models presented in Sects. 3.1 and 3.2.

Energy conditions For $n \neq 0$, (17) leads to

$$R_{ij}u^i u^j = 3(1-n)(nt)^{-2} + 2\beta(nDt)^{\frac{-12\eta_0-6}{n}}, \tag{86}$$

whereas in case of $n = 0$, we get

$$R_{ij}u^i u^j = 3D^2 + 2\beta C_0^{-6} e^{-6(2\eta_0+1)Dt}. \tag{87}$$

We find that $R_{ij}u^i u^j \leq 0$ when $t \geq \frac{1}{nD}[\frac{2\beta}{3D^2(n-1)}]^{\frac{n}{2(6\eta_0+3-n)}}$, which in turn demands $n > 1$. It implies that the Hawking-Penrose condition is satisfied in decelerating models of the universe in the period of time given by $\frac{1}{nD}[\frac{2\beta}{3D^2(n-1)}]^{\frac{n}{2(6\eta_0+3-n)}} \leq t \leq \infty$.

5 Conclusion

In this paper we have studied a spatially homogeneous and anisotropic Bianchi-I space-time with bulk and shear viscosity in general relativity. The field equations have been solved exactly by using a law of variation for the generalized Hubble parameter that yields a constant value of DP. Two exact Bianchi type-I models have been obtained in Sects. 3.1 and 3.2. Expressions for some important cosmological parameters have been obtained for both the models and physical behavior of the models is discussed in detail. In the case of power-law solutions, the universe starts from a singular state whereas exponential solutions follow a non-singular initial state. Due to dissipative processes, the mean anisotropy and the shear of Bianchi-I universe tend to zero very rapidly. It has been observed that shear coefficient (η) plays more important role than bulk coefficient (ξ) in the isotropization process of the universe. These processes represent an effective mechanism for the isotropization of the universe. Therefore it is possible that the isotropy that we observe in the present universe, is a consequence of the viscous effects in the cosmic fluid right from the beginning of evolution of universe. Further if we take $\eta = \xi = 0$, we obtain the perfect fluid models, which have been studied in our previous work [28]. The basic equations of thermodynamics for

Bianchi-I universe have been deduced and thermodynamic aspects of the models have been discussed. Finally, the solutions presented in this paper are new and may be useful for better understanding of the evolution of universe in Bianchi-I space-time with viscous effects in the general theory of gravitation.

References

1. Misner, C.W.: Nature **214**, 40 (1967)
2. Misner, C.W.: Astrophys. J. **151**, 431 (1968)
3. Weinberg, S.: Astrophys. J. **168**, 175 (1971)
4. Nightingale, J.P.: Astrophys. J. **185**, 105 (1973)
5. Murphy, J.L.: Phys. Rev. D **8**, 4231 (1973)
6. Heller, M., Klimek, Z.: Astrophys. Space Sci. **33**, 37 (1975)
7. Belinskii, V.A., Khalatnikov, I.M.: Sov. Phys. JETP **42**, 205 (1976)
8. Banerjee, A., Duttachoudhury, S.B., Sanyal, A.K.: Gen. Relativ. Gravit. **18**, 461 (1986)
9. Banerjee, A., Sanyal, A.K.: Gen. Relativ. Gravit. **20**, 103 (1988)
10. Barrow, J.D.: Nucl. Phys. B **310**, 743 (1988)
11. Grøn, Ø.: Astrophys. Space Sci. **173**, 191 (1990)
12. Gavrilov, V.R., Melnikov, V.N., Triay, R.: Class. Quantum Gravity **14**, 2203 (1997)
13. Arbab, A.I.: Astrophys. Space Sci. **259**, 371 (1998)
14. Beesham, A., Ghosh, S.G., Lombard, R.G.: Gen. Relativ. Gravit. **32**, 471 (2000)
15. Krori, K.D., Mukherjee, A.: Gen. Relativ. Gravit. **32**, 1429 (2000)
16. Saha, B.: Mod. Phys. Lett. A **20**, 2127 (2005)
17. Saha, B.: Astrophys. Space Sci. **312**, 3 (2007)
18. Singh, T., Chaubey, R.: Pramana J. Phys. **68**, 721 (2007)
19. Bali, R., Singh, J.P.: Int. J. Theor. Phys. (2008). doi:10.1007/s10773-008-9764-4
20. Landau, L.D., Lifshitz, E.M.: Fluid Mechanics. Pergamon, New York (1959), p. 47
21. Raychaudhuri, A.K.: Phys. Rev. **98**, 1123 (1955)
22. Berman, M.S.: Nuovo Cim. B **74**, 182 (1983)
23. Berman, M.S., Gomide, F.M.: Gen. Relativ. Gravit. **20**, 191 (1988)
24. Singh, C.P., Kumar, S.: Int. J. Mod. Phys. D **15**, 419 (2006)
25. Singh, C.P., Kumar, S.: Pramana J. Phys. **68**, 707 (2007)
26. Singh, C.P., Kumar, S.: Astrophys. Space Sci. **310**, 31 (2007)
27. Singh, C.P., Kumar, S.: Int. J. Theor. Phys. (2008). doi:10.1007/s10773-008-9752-8
28. Kumar, S., Singh, C.P.: Astrophys. Space Sci. **312**, 57 (2007)
29. Kumar, S., Singh, C.P.: Int. J. Theor. Phys. **47**, 1722 (2008)
30. Kumar, S., Singh, C.P.: Int. J. Mod. Phys. A **23**, 813 (2008)
31. Perlmutter, S., et al.: Astrophys. J. **483**, 565 (1997)
32. Perlmutter, S., et al.: Nature **391**, 51 (1998)
33. Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
34. Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
35. Riess, A.G., et al.: Astron. J. **607**, 665 (2004)
36. Tonry, J.L., et al.: Astrophys. J. **594**, 1 (2003)
37. Knop, R.A., et al.: Astrophys. J. **598**, 102 (2003)
38. John, M.V.: Astrophys. J. **614**, 1 (2004)
39. Vishwakarma, R.G.: Mon. Not. R. Astron. Soc. **345**, 545 (2003)
40. Pradhan, A., Srivastav, S.K., Yadav, M.K.: Astrophys. Space Sci. **298**, 419 (2005)
41. Pradhan, A., Pandey, P.: Astrophys. Space Sci. **301**, 127 (2006)